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Two-loop corrections to the correlator of tensor currents in gluodynamics

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Abstract

Results of evaluating the leading order α_s corrections to the correlator of tensor currents in pure gluodynamics are presented. These corrections to the parton result for the correlator are not large numerically that allows one to use perturbation theory for the analysis of the resonance spectrum within the sum rules method.

Gluonia – hadronic resonances strongly coupled to gauge invariant operators built from gluon fields – are a bright manifestation of the color structure of strong interactions. An experimental discovery of these particles would decisively confirm the validity of QCD as a theory of hadrons, (see e.g. [1]). Theoretically the information on the properties of gluonia can be obtained (besides numerical simulations on the lattice) from the analysis based on the sum rules technique that requires the computation of various correlators of corresponding gluonic interpolating currents within Operator Product Expansion (OPE). Recently an extensive review of gluonia properties has been presented in ref. [2] while Borel sum rules and finite energy sum rules (FESR) were analyzed earlier in [3, 4, 5] (see also [6, 7, 8, 9, 10]).

One of the most striking features of perturbation theory used so far for the analysis of the gluonic current correlators within sum rules approach is a large magnitude of higher order corrections. In the case of scalar and pseudoscalar gluonic currents the next-to-leading order corrections are very large in the standard $\overline{\text{MS}}$ -scheme of renormalization and completely out of control [11] that makes questionable the applicability of the entire analysis based on OPE. Perturbation theory corrections to correlators of gluonic currents with other quantum numbers have not been available yet. In the present letter we fill this gap and report on the results for the leading order correction to the correlator of tensor gluonic currents in pure gluodynamics [12]. Tensor mesons in full QCD were considered in [13, 14, 15].

As for the origin of large corrections in the scalar and pseudoscalar channels, there are strong arguments based on consideration of instanton contributions possible in both these channels that perturbation theory (and OPE) breaks down already at very large scale. This conclusion was made upon considering the magnitude of the power corrections to the gluonic current correlators in the leading order of perturbation theory [3, 4]. Perturbation theory corrections are also large in both channels [11]. On the other hand the interaction of instantons with tensor gluonic currents is thought to be much weaker (no direct instanton contribution to the correlator

is possible because of unsuitable quantum numbers $J^{PC} = 2^{++}$) and one expects that the corrections of perturbation theory are not extremely large and OPE is still applicable at essentially smaller momenta than for the (pseudo)scalar case. The results of the present note explicitly confirm this expectation: the computation shows that perturbation theory corrections to the correlator of tensor gluonic currents are not large and the expansion is valid at smaller scales than in the case of the scalar and pseudoscalar gluonic currents.

To analyze the resonance spectrum in the channel of tensor gluonic mesons one considers a two-point correlator of gluonic operators that have a nonvanishing projection onto the hadronic state with quantum numbers $J^{PC} = 2^{++}$. The gauge invariant interpolating current for the tensor gluonium $j_{\mu\nu}$ is chosen in the following manifest form

$$j_{\mu\nu} = G_{\mu\alpha}^a G_{\alpha\nu}^a + \frac{1}{4} g_{\mu\nu} G^2, \quad a = 1, \dots, N_c^2 - 1 \quad (1)$$

where

$$G_{\mu\nu} = G_{\mu\nu}^a t^a, \quad D_\mu = \partial_\mu - i g_s A_\mu, \quad A_\mu = A_\mu^a t^a, \quad G_{\mu\nu} = [D_\mu, D_\nu],$$

$G_{\mu\nu}$ is the gluon field strength, A_μ is the gluon field, D_μ is the covariant derivative, t^a are the generators of the color gauge symmetry group $SU(N_c)$ normalized by the relation

$$\text{Tr}(t^a t^b) = \frac{1}{2} \delta^{ab}.$$

Here G^2 is a short notation for $G_{\mu\nu}^a G_{\mu\nu}^a \equiv \sum_{a,\mu,\nu} G_{\mu\nu}^a G_{\mu\nu}^a$. The current $j_{\mu\nu}$ coincides with the energy-momentum tensor of pure gluodynamics. It conserves due to equations of motion for the gluon fields, is symmetric and traceless at the tree level. These properties lead to some linear constraints on the components of the tensor $j_{\mu\nu}$ that effectively kill the superfluous components of the general two-index tensor in four-dimensional space-time and keep just the necessary number of components to describe five polarization states of a massive meson with spin 2 in four-dimensional space-time. Radiative corrections is known to destroy this perfect picture and lead

to the trace anomaly of the form [16, 17, 18, 19]

$$j_\mu^\mu = \frac{\beta(\alpha_s)}{2\alpha_s} G^2 \quad (2)$$

where $\beta(\alpha_s)$ is the standard renormalization group β -function describing the evolution of the running coupling constant. While the rigorous proof of the relation for the trace anomaly requires rather delicate definitions of the quantities entering eq. (2), there is a simple mnemonic rule to recover the proper normalization of the right hand side of it. In D -dimensional space-time (with $D = 4 - 2\varepsilon$) one formally finds from eq. (1) with $g_\mu^\mu = D$

$$j_\mu^\mu = \frac{D-4}{4} G^2 = -\frac{\varepsilon}{2} G^2 . \quad (3)$$

Recalling that the D -dimensional β_ε -function is given by the expression $\beta_\varepsilon(\alpha_s) = -\varepsilon\alpha_s + O(\alpha_s^2)$ and substituting $\varepsilon = -\beta_\varepsilon(\alpha_s)/\alpha_s$ into the right hand side of eq. (3) one reproduces the correctly normalized expression in the right hand side of eq. (2) noticing that $\lim_{\varepsilon \rightarrow 0} \beta_\varepsilon(\alpha_s) = \beta(\alpha_s)$.

The appearance of nonvanishing trace of the gluonic operator in eq. (1) means that the operator has a nonvanishing projection onto the scalar hadronic states as well. In the language of Green's functions this means that the correlator

$$T_{\mu\nu,\alpha\beta}(q) = i \int dx e^{iqx} \langle T j_{\mu\nu}(x) j_{\alpha\beta}(0) \rangle \quad (4)$$

gets contributions not only from the tensor mesons with $J^{PC} = 2^{++}$ but also from the states with quantum numbers $J^{PC} = 0^{++}$, or the scalar gluonium. Therefore the correlator (4) is not described by the single scalar function when radiative corrections are included. The most general tensor decomposition of the correlator (4) has the form

$$T_{\mu\nu,\alpha\beta}(q) = \eta_{\mu\nu,\alpha\beta}(q) T(q^2) + f_{\mu\nu,\alpha\beta}(q) T_S(q^2) \quad (5)$$

where the tensor object $\eta_{\mu\nu,\alpha\beta}(q)$ is defined through the elementary transverse tensors

$$\eta_{\mu\nu} = q_\mu q_\nu - q^2 g_{\mu\nu}$$

by the expression

$$\eta_{\mu\nu,\alpha\beta}(q) = \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \frac{2}{3}\eta_{\mu\nu}\eta_{\alpha\beta}. \quad (6)$$

The quantity $\eta_{\mu\nu,\alpha\beta}(q)$ is a density (polarization) matrix of a particle with spin 2 which determines the structure of its propagator in momentum space. It has the properties

$$q^\mu\eta_{\mu\nu,\alpha\beta}(q) = 0, \quad q^\alpha\eta_{\mu\nu,\alpha\beta}(q) = 0, \quad \eta_{\mu\mu,\alpha\beta}(q) = 0, \quad \eta_{\mu\nu,\alpha\alpha}(q) = 0. \quad (7)$$

Note that the last two relations in (7) are valid only in four-dimensional space-time. The proper tensor in D -dimensional space-time, necessary for computation within dimensional regularization, reads

$$\eta_{\mu\nu,\alpha\beta}^D(q) = \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \frac{2}{D-1}\eta_{\mu\nu}\eta_{\alpha\beta}. \quad (8)$$

It has a vanishing trace and is orthogonal to the quantity $f_{\mu\nu,\alpha\beta}(q)$. However, both are equivalent to the order of perturbation theory we work in the present paper, and the difference between eq. (6) and eq. (8) is inessential. The quantity $\eta_{\mu\nu,\alpha\beta}(q)$ is symmetric in both pairs of indices $(\mu\nu)$ and $(\alpha\beta)$. The second tensor object entering eq. (5)

$$f_{\mu\nu,\alpha\beta}(q) = \eta_{\mu\nu}\eta_{\alpha\beta}$$

is the tensor structure related to the contribution of scalar particles. In a general D -dimensional space-time the tensors $f_{\mu\nu,\alpha\beta}(q)$ and $\eta_{\mu\nu,\alpha\beta}(q)$ are not orthogonal to each other. The perturbation theory expansion of the function $T_S(q^2)$ starts with the terms of order α_s^2 (the nonvanishing imaginary part) and is negligible in the considered (next-to-leading) order in α_s to which we limit ourselves in the present paper. Nonvanishing term of this tensorial structure emerges due to the trace anomaly. Thus, up to the terms of order $O(\alpha_s^2)$ the correlator (4) is determined by the single scalar function $T(q^2)$ related to the contribution of tensor gluonia only. The anomalous dimension of the current $j_{\mu\nu}$ vanishes that makes the function $T(q^2)$ invariant with regard to the renormalization group transformations.

The results of direct computations of the function $T(q^2)$ are as follows. The leading order contribution to the function $T(q^2)$ is well known [4]. Within dimensional regularization with $D = 4 - 2\varepsilon$ being the space-time dimensionality it has the form

$$\frac{N_c^2 - 1}{(4\pi)^2} \frac{1}{10\varepsilon} \left(\frac{\mu^2}{Q^2} \right)^\varepsilon G(\varepsilon), \quad Q^2 = -q^2. \quad (9)$$

The quantity $G(\varepsilon)$ is related to the particular definition of the integration measure in D -dimensional momentum space-time and has the series expansion $G(\varepsilon) = 1 + O(\varepsilon)$ at small ε [20], μ is the t'Hooft mass of dimensional regularization. The factor $N_c^2 - 1$ counts the number of gluons (or partons at this level of computation within perturbation theory) of the color group $SU(N_c)$ propagating in the single loop to which the correlator reduces in this order of perturbation theory.

For the amplitude $T(Q^2)$ with account for the two-loop perturbation theory corrections one finds

$$T(Q^2) = \frac{N_c^2 - 1}{10(4\pi)^2} \frac{1}{\varepsilon} \left(\frac{\mu^2}{Q^2} \right)^\varepsilon G(\varepsilon) \left(1 + \frac{\alpha_s}{4\pi} N_c \left(-\frac{10}{9} \right) \left(\frac{\mu^2}{Q^2} \right)^\varepsilon G(\varepsilon) \right). \quad (10)$$

This expression represents the main result of the present note. The coefficient of the leading order correction to the correlator was first computed in ref. [12] and later confirmed by independent computation [21]. The basic technique of the computation is described in detail in ref. [11] where the scalar and pseudoscalar cases were considered. Diagrams are the same in the tensor case. Explicit expressions for the vertices and results for individual diagrams can be found in ref. [12]. The only complication in the present case is the tensor structure of diagrams. There are several different tensor projectors which can convert the necessary expressions into the scalar form. Some related integrals can be found in ref. [22] where the correlator of quark currents with spin n was considered. Poles of order ε^2 which are possible at the two-loop level (in the α_s order) and actually present in the expressions for particular diagrams cancel in expression (10) as it should be because of renormalization group invariance of the current $j_{\mu\nu}$. For the corresponding D -function (a derivative of the amplitude $T(Q^2)$) which is multiplicatively renormalized

one has

$$D_T(Q^2) = -Q^2 \frac{d}{dQ^2} T(Q^2) = \frac{N_c^2 - 1}{160\pi^2} \left(1 - \frac{\alpha_s}{4\pi} N_c \frac{20}{9} \right). \quad (11)$$

The coefficient of α_s in the last expression is independent of the renormalization scheme used for the calculation of the amplitude $T(Q^2)$; it is also seen in the fact that one has no need to fix a precise definition of the integration measure: the quantity $G(\varepsilon)$ enters the final answer only as a factor $G(0) = 1$. Evaluating the numerical value for this coefficient is the real content of two-loop calculations of the correlator (4) and the amplitude $T(q^2)$ in eq. (5) in particular.

Numerically, for the standard gauge group with $N_c = 3$ one has

$$D_T(Q^2) = \frac{1}{20\pi^2} \left(1 - \frac{5}{3} \frac{\alpha_s(Q^2)}{\pi} \right). \quad (12)$$

Thus, the next-to-leading order correction to the correlator of tensor gluonic currents is not large and the perturbation theory expansion is quite well convergent numerically even at $\alpha_s \approx 0.3$ that corresponds to the scale of ordinary hadrons. The remarkable feature of this correction is its sign. In most cases α_s corrections are positive while for the tensor correlator of currents (1) it is negative. Inclusion fermions has a two-fold effect: loop corrections through the gluon propagator and the mixing with fermionic operators at the tree level. The former effect is trivial (explicit results can be found in ref. [12]) while the latter was discussed in the literature.

The smallness of the first order correction to the observables which are renormalization group invariant at the parton level is a rather general feature of hadron phenomenology. The most famous example is the total cross section of e^+e^- annihilation into hadrons where the perturbation theory corrections in $\overline{\text{MS}}$ -scheme are not large, with the leading order correction explicitly given by

$$\sigma_{tot}(e^+e^- \rightarrow \text{hadrons}) \sim 1 + \frac{\alpha_s}{\pi} + \dots$$

However, in cases when corrections depend on the definition of the coupling constant in the considered channels (as in the case of the scalar or pseudoscalar gluonium)

they can be rather large in $\overline{\text{MS}}$ -scheme. For instance, for the pseudoscalar gluonium with the interpolating operator

$$j_{PS} = \alpha_s G \tilde{G} = \alpha_s \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} G_{\mu\nu}^a G_{\alpha\beta}^a$$

one finds [11]

$$D_{PS}(Q^2) = \frac{2\alpha_s^2(Q^2)}{\pi^2} \left(1 + \frac{97}{4} \frac{\alpha_s(Q^2)}{\pi} \right). \quad (13)$$

The correction of order α_s in eq. (13) is much larger than that in eq. (12) and makes perturbation theory inapplicable at momenta of the order of the standard values of hadronic resonance masses.

It is worth noticing that the D -functions are defined in Euclidean domain while the physical spectrum requires the correlators to be evaluated on the physical cut. For two-point correlators the analytic properties are well established and the analytic continuation can be done in all order in α_s (e.g. [23]) which can, however, change the effective numerical magnitude of the total correction in concrete applications.

Whether the magnitude of corrections reflects the physical situation – contribution of instantons and early breakdown of perturbation theory – is still an open question which urgently requires further investigation.

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